

Effect of Electron Interaction on Statistics of Conductance Oscillations in Open Quantum Dots: Does the Dephasing Time Saturate?

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We perform self-consistent quantum transport calculations in open quantum dots taking into account the effect of electron interaction. We demonstrate that in the regime of the ultralow temperatures $2\pi k_B T \lesssim \Delta$ (Δ being the mean level spacing), the electron interaction strongly affects the conductance oscillations and their statistics leading to a drastic deviation from the corresponding predictions for noninteracting electrons. In particular, it causes smearing of conductance oscillations, which is similar to the effect of temperature or inelastic scattering. For $2\pi k_B T \gtrsim \Delta$ the influence of electron interaction on the conductance becomes strongly diminished. Our calculations (that are free from phenomenological parameters of the theory) are in good quantitative agreement with the observed ultralow temperature statistics (Huibers *et al.*, Phys. Rev. Lett. **81**, 1917 (1998)). Our findings question a conventional interpretation of the ultralow temperature saturation of the coherence time in open dots which is based on the noninteracting theories where the electron interaction is neglected and the agreement with the experiment is achieved by introducing additional phenomenological channels of dephasing.

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Introduction. Decoherence of quantum states due to interaction with an environment represents one of the fundamental phenomena in quantum physics. In low-dimensional semiconductor structures such as quantum dots, wires, antidots the decoherence processes are central to electronic transport and spin/charge manipulation. A temperature dependence of the phase coherence time in open quantum dots τ_φ has been a focus of significant experimental activity during the past decade[1, 2, 3, 4, 5, 6, 7, 8]. The experiments show that τ_φ increases as the temperature T decreases following a dependence $\tau_\varphi \sim T^{-\gamma}$, where $1 \lesssim \gamma \lesssim 2$. This behavior indicates that several mechanisms might be simultaneously responsible for the electron decoherence, including electron scattering with a large energy transfer[9] (leading to $\tau_\varphi \sim T^{-2}$ dependence), and a small energy-transfer (Nyquist) scattering[10] (giving a rate $\tau_\varphi \sim T^{-1}$). Surprisingly, practically all experiments report a remarkable effect of a saturation of the phase coherence time at ultralow temperatures $T \lesssim 100$ mK. The origin of this effect is not understood and at present no theory is available addressing the electron decoherence in confined ballistic systems. It should be noted that a similar effect of a saturation of the phase coherence time is also found in nanoscaled metallic wires and the origin of this saturations also remains open and highly debated[11].

An experimental determination of the dephasing time τ_φ is typically based on predictions of the random matrix theory (RMT)[12] for the statistics for quantum transport such as the mean and variance of the conductance oscillations, the weak localization corrections, the probability distribution of conductance and others[12, 13, 14]. The RMT is essentially noninteracting theory relying on

a one-electron description of quantum transport. In order to fit the experimental data, the dephasing time τ_φ is included as a phenomenological parameter of the theory typically within a Büttiker's fictitious voltage probe or as an imaginary potential in the Hamiltonian[12, 14]. A deviation of the experimental data from the predictions of a purely coherent model of noninteracting electrons is then attributed to inelastic scattering due to dephasing which is extracted using τ_φ as a fitting parameter.

How does the electron interaction affect the conductance oscillations in the open dots? This question was posed in several theoretical studies with somehow conflicting conclusions[15, 16, 17, 18]. For example, Brouwer and Aleiner[15] argued that the Coulomb interactions enhance the weak localization and increase conductance fluctuations, whereas Brouwer *et al.*[16] questioned these conclusions. None of the above studies however addressed the problem of the low-temperature saturation of the coherence time. In the present paper we, based on the first-principle self-consistent quantum transport calculations, study the effect of the electron interaction on the probability distribution of the conductance, $P(G)$, in open dots. We demonstrate that for sufficiently high temperatures (when the transport energy window exceeds the mean level spacing, $2\pi k_B T \gtrsim \Delta$, the corresponding distributions $P(G)$ for interacting and noninteracting electrons are practically the same. However, in the opposite limit of ultralow temperatures, $2\pi k_B T \lesssim \Delta$, the distributions of $P(G)$ are strikingly different for noninteracting and interacting electrons. We compare our calculated statistics for interacting electrons with corresponding experimental results of Huibers *et al.* [5] and find a good quantitative agreement. Our results therefore strongly indicate that a deviation of the experimental

data from the RMT predictions in the regime of ultralow temperatures can be accounted for by the electron interaction alone without introducing additional channels of the inelastic scattering. Our findings thus question the conclusion concerning the saturation of the τ_φ in open dots which is obtained neglecting electron interaction and under the assumption that the above deviation is due to the inelastic scattering only.

Model. We consider an open quantum dot defined by split-gates in GaAs heterostructure, see Fig. 1. The Hamiltonian of the whole system (the dot + the semi-infinite leads) can be written in the form $H = H_0 + V(\mathbf{r})$, where $H_0 = -\frac{\hbar^2}{2m^*} \left\{ \left(\frac{\partial}{\partial x} - \frac{e i B y}{\hbar} \right)^2 + \frac{\partial^2}{\partial y^2} \right\}$ is the kinetic energy in the Landau gauge, and the total confining potential $V(\mathbf{r}) = V_{conf}(\mathbf{r}) + V_H(\mathbf{r})$ is the sum of the electrostatic confinement (including contributions from the top gates, the donor layer and the Schottky barrier), and the Hartree potential, see [18, 19] for details

$$V_H(\mathbf{r}) = \frac{e^2}{4\pi\epsilon_0\epsilon_r} \int d\mathbf{r}' n(\mathbf{r}') \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} - \frac{1}{\sqrt{|\mathbf{r} - \mathbf{r}'|^2 + 4b^2}} \right), \quad (1)$$

where $n(\mathbf{r})$ is the electron density, the second term corresponds to the mirror charges situated at the distance b from the surface, and $\epsilon_r = 12.9$ is the dielectric constant of GaAs. Note that the dot and the leads are treated on the same footing, e.g. the Coulomb interaction and the magnetic field are included both in the lead and in the dot regions. We consider the spinless electrons because in relatively large dots as those studied here the electrons are spin degenerate[20, 21]. We also neglect the exchange and correlation effects, which have been shown to affect the calculated conductance only marginally[18].

To outline the role of the electron interaction in the conductance of open quantum dots we also calculate magnetoconductance in the Thomas-Fermi (TF) approximation. In this approximation the self-consistent electron density is given by the standard TF equation, $\frac{\pi\hbar^2}{m^*} n(x, y) + V_{conf}(\mathbf{r}) + V_H(\mathbf{r}) = E_F$. The electron density and the total confining potential calculated within the TF approximation do not capture quantum-mechanical quantization of the electron motion and the resonant energy structure in the dot. The utilization of the TF approximation for modeling of the magnetotransport in open system is therefore conceptually equivalent to noninteracting one-electron transport calculations, where, however, the total confinement is given by a smooth realistic potential.

The magnetoconductance through the quantum dot in the linear response regime is given by the Landauer formula, $G = -\frac{2e^2}{h} \int dE T(E) \frac{\partial f_{FD}(E - E_F)}{\partial E}$. A detailed description of the self-consistent conductance calculation (as well as the validity and applicability of the method and the Hamiltonian) are given in our previous publications [18, 19]. Note that the present approach corresponds to the “first principle” magnetoconductance cal-

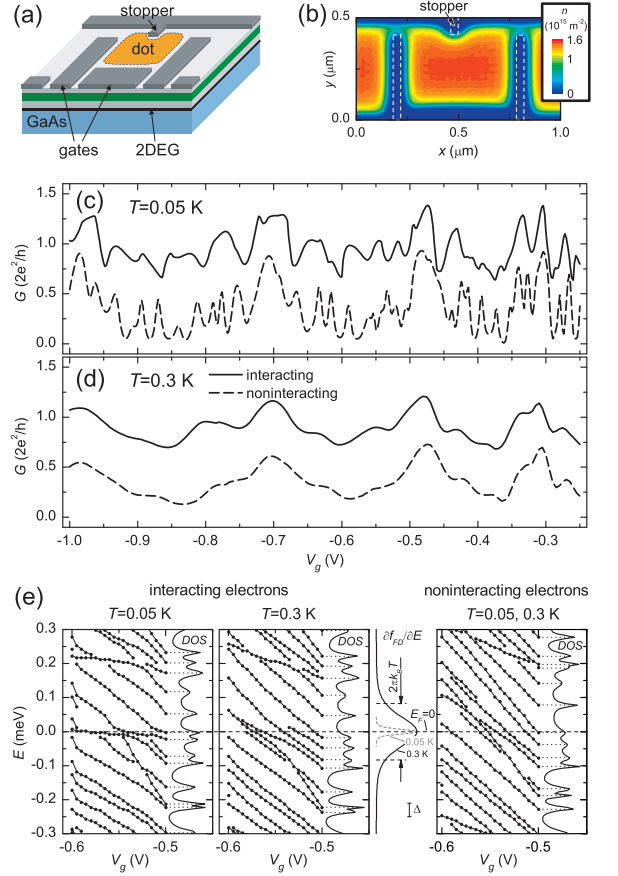


FIG. 1: (Color online). (a) A schematic layout of a split-gate open quantum dot defined in a GaAs heterostructure. The geometrical size of the dot is $660 \times 520 \text{ nm}$, the width of the leads is 540 nm . The widths of the cap, donor and spacer layers are 14 nm , 36 nm and 10 nm respectively, the donor concentration is $0.6 \cdot 10^{24} \text{ m}^{-3}$. (b) A representative self-consistent electron density in the dot (note that the densities for the interacting (Hartree) and noninteracting (TF) electrons are not distinguishable on the scale of the figure). White dashed lines indicate a geometry of the metallic gates. (c),(d) The calculated conductance of the dot for interacting (Hartree) and noninteracting (TF) electrons as a function of the gate voltage V_g for (a) $T = 50 \text{ mK}$ and (b) $T = 300 \text{ mK}$; $B = 20 \text{ mT}$. [The conductance curves for interacting electrons are shifted by e^2/h .] (e) Resonant energy structure (i.e. positions of the peaks in the DOS) as a function of the V_g for different temperatures; A calculated DOS is shown for $V_g = -0.5 \text{ V}$; the Fermi energy is set $E_F = 0$. [Note that the resonant energy structure for noninteracting electrons is practically undistinguishable for the given temperatures]. Inset shows the derivative of the Fermi-Dirac distribution function for $T = 0.05 \text{ K}$ and 0.3 K and the transport window $2\pi k_B T$.

ulation (within the effective mass approximation), that starts from a geometrical layout of the device, is free from phenomenological parameters and not relying on model Hamiltonians whose validity is poorly controlled.

Results and discussions. Figure 1(c) shows the conductance of the open quantum dot calculated in the

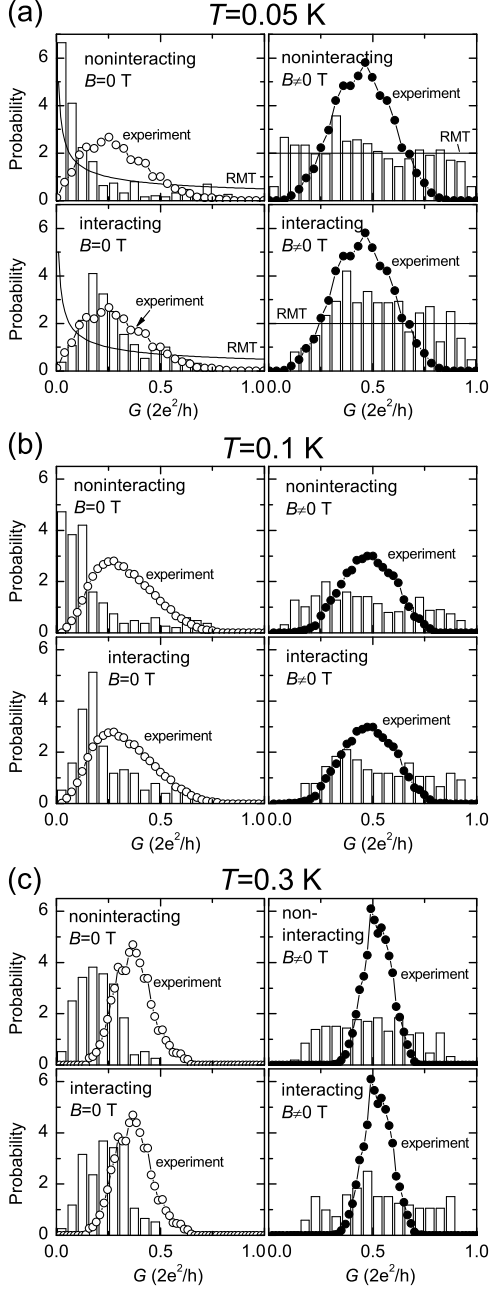


FIG. 2: Probability distribution of the conductance $P(G)$ for interacting and noninteracting electrons for different temperatures for the cases of the time-reversed symmetry ($\beta = 1$) and the broken time-reversed symmetry ($\beta = 2$). The experimental data is adapted from Ref. [5]. Solid lines in (a) corresponds to the predictions of the RMT ($T = 0$, no dephasing).

Hartree and TF approximations (interacting and noninteracting electrons respectively) for $T = 50\text{mK}$. The parameters of the dot are indicated in Fig. 1 and are chosen close to those studied experimentally by Huibers *et al.* [5]. All the results discussed in this paper correspond to one propagating mode in the quantum point contact

openings. The striking difference between the conductance curves is clearly manifested in a strong suppression of the high frequency components of the oscillations for the interacting electrons in comparison to the noninteracting case. *Thus, the electron interaction causes an apparent smearing of the conductance oscillations, which is similar to the effect of the temperature or inelastic scattering.* This smearing of oscillations is caused by the pinning of resonant levels to the Fermi energy in the vicinity of resonances[18]. This is illustrated in Fig. 1 (e) which shows an evolution of the peak position of the resonant energy levels. In the vicinity of the resonances the DOS of the dot is enhanced such that electrons with the energies close to E_F can easily screen the external potential. This leads to the “metallic” behavior of the system when the electron density in the dot can be easily redistributed to keep the potential constant. As a result, in the vicinity of a resonance the system only weakly responds to the external perturbation (change of a gate voltage, magnetic field, etc.), i.e. the resonant levels becomes pinned to the Fermi energy (see Ref. [18] for a detailed discussion of the pinning effect). For noninteracting electrons the non-linear screening and hence the pinning effect are absent, such that the successive dot states sweep past the Fermi level in a linear fashion, see Fig. 1 (e).

The pinning of resonant levels drastically affects the conductance probability distribution $P(G)$. Figure 2(a) shows $P(G)$ calculated for interacting and noninteracting electrons for the cases of a time-reversal symmetry, $\beta = 1$ ($B = 0$) and a broken time-reversal symmetry, $\beta = 2$, ($B \neq 0$) for $T = 50\text{mK}$. The time-reversal symmetry is broken by application of a magnetic field $B \gtrsim \phi_0/A$, where $\phi_0 = h/e$ is the flux quantum and A is the dot area (typically, $B \sim 20 - 40\text{mT}$). Figure 2 (a) shows that the statistics of the conductance distribution $P(G)$ for the case of noninteracting electrons closely follows the corresponding RMT predictions for $\tau_\varphi = 0$ and $T = 0$ [12, 13] both for $\beta = 1$ and $\beta = 2$. At the same time, the statistics for the interacting electrons are strikingly different from those for the noninteracting case. Thus, *due to the effect of the electron interaction, the ultra-low temperature statistics of the conductance oscillations of quantum dots are not described by the RMT.*

As the temperature increases, the difference between the conductances $G = G(V_g)$ as well as between the corresponding conductance distributions $P(G)$ for interacting and noninteracting electrons diminishes, see Fig. 2 (b) ($T = 100\text{mK}$). For sufficiently high temperature this difference disappears, see Figs. 1 (d), 2 (c) ($T = 300\text{mK}$). The reason for that is that the temperature strongly reduces the effect of resonant level pinning. Indeed, when the transport energy window, $\sim 2\pi k_B T$, (determined by the condition when the derivative of the Fermi-Dirac distribution is distinct from zero, see Fig. 1 (e)) exceeds the mean level spacing $\Delta = \frac{2\pi\hbar^2}{m^*A}$ (A being the dot area) the conductance is mediated by several levels. As a re-

sult, several levels always contribute to screening at the same time and hence the screening efficiency of the dot is affected very little when a gate voltage or a magnetic field are varied. A quenching of the pinning for temperatures $2\pi k_B T \gtrsim \Delta$ due to suppression of the resonant level screening is illustrated in Fig. 1 (e) ($T = 300\text{mK}$). Note that for the dot under consideration the condition $2\pi k_B T = \Delta$ corresponds to $T \approx 100\text{mK}$. Thus, for $2\pi k_B T \gtrsim \Delta$ the effect of electron interaction on the conductance is strongly suppressed such that the conductance and their probability distributions for interacting and noninteracting electrons are practically the same.

Let us now compare our results with available experimental data. The probability distribution $P(G)$ in open quantum dots with one propagating channel in the leads was studied by Huiberts *et al.* [5]. Figure 2 (a) shows that in the regime of the ultralow temperatures, $T = 50\text{mK}$, the calculated conductance statistics for interacting electrons agree quite well with the corresponding experimental distribution $P(G)$ both for $\beta = 1$ and $\beta = 2$. The measured conductance distribution $P(G)$ in Ref. 5 was well described by the RMT predictions where the inelastic scattering was introduced using τ_φ as a fitting parameter. Our results, instead, demonstrate, that once the electron interaction is accounted for, the agreement with the experiment for $2\pi k_B T \lesssim \Delta$ is achieved without assuming additional inelastic scattering channels. We thus conclude that for the regime of ultralow temperatures the experimentally inferred value of τ_φ might be greatly underestimated which implies that the dephasing time does not saturate. As the temperature increases, the calculated conductance distribution starts to deviated from the experimental statistics, see Fig 2 (b),(c). As discussed above, for the temperature $2\pi k_B T \gtrsim \Delta$ the electron interaction practically does not affect the conductance oscillations and their statistics. Thus, for $2\pi k_B T \gtrsim \Delta$ the difference between the calculated and experimental statistics can be attributed to the effect of dephasing. Our criterium for the transition temperature $2\pi k_B T \sim \Delta$ is consistent with the findings reported by Bird *et al.* [2, 7] and Clarke *et al.*[3] who find a saturation behavior of τ_φ at transition temperatures T_{onset} near the mean-level spacing. A relation between T_{onset} and Δ was also discussed by Hackens *et al.* [8]. However, some experiments[6] do not show a clear relation between T_{onset} and Δ , such that more systematic studies are needed in order to prove the connection between T_{onset} and Δ .

We stress that our calculations are performed for purely coherent electrons. The dephasing effects can be easily included in our model phenomenologically through an imaginary potential in the Hamiltonian[12]. We do not provide a systematic fit of the experiment simply because of a computation burden related to this task: each point on the conductance plot requires up to one hour of a processor time. We however note that such a fit is

outside the scope of our study, where we focuss on the role of the electron interaction in a regime of the ultralow temperatures, $2\pi k_B T \lesssim \Delta$.

The findings reported in this paper outline importance of the “first-principle” self-consistent quantum transport calculations for open quantum dots. Indeed, accounting for both global electrostatics through the Hartree potential, Eq. (1), and the quantum mechanical quantization in a self-consistent way is essential for revealing of the pinning effect that causes a drastic difference in the conductance of the interacting and noninteracting electrons. Note that this effect would not be captured in the approach utilizing model Hamiltonians (like those of Ref. [15, 16] where the electron interaction is accounted thought the classical capacitance charging).

To conclude, we demonstrate that for ultralow temperatures $2\pi k_B T \lesssim \Delta$ the electron interaction drastically changes the statistics of the conductance oscillations in open dots leading to a significant departure from the conventional RMT description of noninteracting electrons. Our results demonstrate that the deviation of the observed statistics at ultralow temperatures from the RMT predictions can be accounted for by the electron interaction alone, such that a conclusion of the dephasing time saturation based on noninteracting electron picture should be revised.

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